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Neutrinos, their partners, and unification

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Abstract. Efforts to unify group-theoretically the standard-model gauge interactions with the generation structure of fermions and their mirror partners should be accompanied by the unification of the corresponding gauge couplings. In this paper, the possibility of such a unification is studied, and conclusions on possible symmetry-breaking channels and scales as well as on the fermion content of the theory are drawn. The breaking of some of the symmetries allows various Majorana masses for neutrinos and their mirror partners, so these are studied next. Implications for neutrino mixings and mass hierarchies in connection with recent experimental results, as well as for electroweak precision tests, are then discussed.

1 Introduction

The quantum numbers of the known fermions under the standard-model gauge structure allow their partial classification under the fundamental representations of the corresponding symmetry groups. This motivates efforts to complete this classification by studying unifying symmetry groups large enough to accommodate all the fundamental-particle generations which have been observed so far. Apart from the purely theoretical interest in such a possibility, quests for such a unification usually lead to predictions of the existence of new particles, such as extra fermions and gauge bosons [1–3]. In particular, the new fermions are usually referred to as the "mirror partners" of the standard-model fermions.

Since there are currently several theoretical and possibly also some experimental indications hinting at the existence of physics beyond the standard model at scales on the order of 1 TeV [1], it is worthwhile to investigate whether extensions of particle theories in a direction compatible with generation unification could be related to these indirect indications at TeV-energy scales. Furthermore, in view of the fact that accelerators designed to operate in the next decade plan to cover such high energies, it is quite important to investigate their discovery potential by producing directly the particles predicted by the aforementioned extensions.

Before embarking on such a detailed production-and-decay study, however, one should first check the internal consistency of the proposed theories and their compatibility with current experimental constraints. A first effort to reproduce the observed charged-fermion mass hierarchy, the quark-mixing matrix elements and the weak scale while staying in agreement with the electroweak precision data within such a framework was recently presented [1]. The purpose of the present work is to tackle some related, equally important open issues.

One of these issues involves the calculation of the evolution of the gauge couplings to very high energies, to determine if there is a sequence of symmetry-breakings consistent with the unification picture which motivated the proposed extension in the first place. In all cases discussed, the symmetries in question are taken to break spontaneously, and getting into the details of the breaking mechanism, e.g., its being of dynamical or fundamental nature, or the transformation properties of its nonzero vacuum expectation value, is avoided, because this usually involves a high degree of arbitrariness and speculation in an area of no phenomenological input.

Since the energy scales of these breakings could be associated with the lightness of the standard-model neutrinos via the seesaw mechanism, the question of neutrino masses and mixings left open in [1] has to be studied next. This also allows the calculation of novel "oblique" contributions to the electroweak parameters that are due to the possible Majorana nature of the mirror neutrinos. An effort to address these different but closely related issues follows next.

2 Coupling unification

2.1 Preliminary considerations

The starting point of the discussion could be either of the unification gauge groups $E_{8_1} \times E_{8_2}$ or $SO(16)_1 \times SO(16)_2$ without change in the final results, with gauge couplings g_1 and g_2 corresponding to the groups with subscripts 1 and 2 respectively. These symmetries are taken to break at the unification scale Λ_{GUT} down to $SO(10)_1 \times SU(4)_{1\mathrm{G}} \times SO(10)_2 \times SU(4)_{2\mathrm{G}}$. The fermions and mirror fermions of interest transform under the above groups as $(\mathbf{16},\ \bar{\mathbf{4}},\ \mathbf{1},\ \mathbf{1})$ and $(\mathbf{1},\ \mathbf{1},\ \bar{\mathbf{16}},\ \mathbf{4})$ respectively.

It is imagined next that around the same unification scale, $SO(10)_1 \times SO(10)_2$ breaks to its diagonal subgroup $SO(10)_\mathrm{D}$, which has accordingly a gauge coupling at that scale equal to

$$g = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}. \tag{1}$$

This in turn is taken to break again at Λ_GUT into its maximal subgroup $SU(4)_\mathrm{PS} \times SU(2)_\mathrm{L} \times SU(2)_\mathrm{R}$. In addition, $SU(4)_{1\mathrm{G}} \times SU(4)_{2\mathrm{G}}$ could be taken to break down at the same scale to either of the three groups $SU(3)_{2\mathrm{G}}$, $SU(3)_{2\mathrm{G}} \times U(1)_\mathrm{G}$, or $SU(4)_{2\mathrm{G}}$. Under the resulting group $SU(4)_\mathrm{PS} \times SU(2)_\mathrm{L} \times SU(2)_\mathrm{R} \times SU(3)_{2\mathrm{G}}$, for instance, the standard model-type fermions transform into four copies ("generations") of $(\mathbf{4}, \mathbf{2}, \mathbf{1}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}, \mathbf{1})$, and the mirror fermions to $(\mathbf{4}, \mathbf{1}, \mathbf{2}, \mathbf{3})+(\mathbf{4}, \mathbf{1}, \mathbf{2}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}, \mathbf{3}) + (\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}, \mathbf{1})$. The other two possibilities will be discussed in Sect. 2.4. In all these cases, it is assumed that the fourth-generation standard model-type fermions pair up with their mirror partners, acquiring gauge-invariant masses on the order of the $SU(4)_{2\mathrm{G}}$-breaking scale. (This is Λ_GUT for the first two cases and about 1 TeV for the third case, as will be seen in Sect. 2.4.)

One first notes that the only way to unify the generation-group coupling with the other gauge couplings is to satisfy the relation $g_1 \gg g_2$ at Λ_GUT, because then the common unification coupling is $g \approx g_2$ according to (1). Therefore, the generation group $SU(4)_{1\mathrm{G}}$, which is taken to break completely at Λ_GUT, is strongly coupled at that scale. The situation with generation group $SU(3)_{2\mathrm{G}}$ with coupling g_2 is first investigated. The basis of the analysis of the gauge-coupling renormalization that follows is more of a qualitative nature and limited to the one-loop β function, because of the many uncertainties of the dynamics influencing the running of these couplings. These are mainly due not only to our ignorance of the exact masses of the mirror fermions and of the type of new physics needed to break the gauge groups involved in this picture, which could be Higgs particles in various presently unpredictable representations, but also to the possible existence of supersymmetric partners of the standard-model fermions and to threshold effects near the unification scale.

These uncertainties lead one to take all the mirror fermions to have the same mass Λ_M at around 1 TeV for simplicity, since the coupling unification is found to be quite insensitive to this scale anyway. Below Λ_M, the couplings evolve as in the standard model. Above that scale, one has to take the mirror fermions into account. It is then assumed that there exists a "desert" between Λ_M and the Pati–Salam scale Λ_PS where $SU(4)_\mathrm{PS}$ is broken, with no new dynamics or particles able to influence the evolution of the gauge couplings with energy.

The β function describing the evolution of the gauge coupling g of an $SU(N)$ group with N_f fermion N-plets with respect to momentum p is given by

$$\beta \equiv \frac{\mathrm{d}g}{\mathrm{d}\ln(p/p_0)} = -\frac{g^3}{48\pi^2}(11N - 2N_\mathrm{f} + r), \tag{2}$$

where r stands for higher-than-one loop corrections, and p_0 is some reference scale. If the same fermions transform also under the fundamental representation of another unitary gauge group $SU(N')$ with coupling g' much larger than g, the quantity r at two loops is approximately given by [4]

$$r = \frac{g'^{\,2}(N'^{\,2} - 1)}{32\pi^2}. \tag{3}$$

Therefore, when the $SU(3)_{2\mathrm{G}}$ interactions become strong at around 2 TeV and break $SU(2)_\mathrm{L} \times U(1)_\mathrm{Y}$ dynamically by an effective Higgs mechanism induced by fermion condensates [1], the corresponding fine-structure constant is $\alpha_\mathrm{G} \approx 1$. One therefore gets $r \approx 0.3$, which is still much smaller than the one-loop contribution to the other couplings, even for the smaller groups considered, e.g., $SU(2)_\mathrm{L}$ or $U(1)_\mathrm{Y}$ (the influence of the other couplings to each other is of course even more negligible due to their smallness). In addition, since $SU(3)_{2\mathrm{G}}$ is taken to break just after it becomes strong [1]; it has a rather limited energy region where it can influence substantially the β functions of interest, so large deviations from the one-loop renormalization of the rest of the gauge couplings are not expected. This issue is investigated further by presenting a particular example in Sect. 2.3.

Moreover, a fundamental Higgs mechanism for breaking $SU(3)_{2\mathrm{G}}$ is avoided by the evocation of the mechanism conjectured in the appendix of [1]. In any case, a minimal fundamental Higgs mechanism breaking the generation symmetry, apart from all the naturalness problems it carries with it, would make the corresponding gauge coupling run slightly slower. The generation-coupling unification with the rest of the gauge couplings at Λ_GUT would then still be achievable by the slight lowering of the maximal value this coupling reaches before $SU(3)_{2\mathrm{G}}$ breaks and/or the lowering of the mirror-mass scale Λ_M. An effort to estimate the energy scales entering this problem without fundamental scalars is presented in the next subsection.

However simple, the approach adopted allows us to draw general conclusions, that do not depend on particular details, about the way the unification groups break down to the standard-model gauge structure. It must be stressed, nevertheless, that the class of symmetry-breaking channels of interest here has an additional degree of freedom compared to the usual and the supersymmetric unifications: the Pati–Salam symmetry-breaking scale Λ_PS. This can be in most cases slightly adjusted to allow unification of couplings even after the correct inclusion of these corrections, unless one introduces unnaturally large Higgs sectors to break the gauge symmetries. The results that follow should therefore be seen not as exact predictions, but rather as order-of-magnitude estimates.

2.2 Calculation of Λ_GUT and proton lifetime

The analysis presented here is based on different alternative breakings of the gauge symmetry $SU(4)_\mathrm{PS}$, since *a priori* there is no obvious reason to expect a specific breaking channel. The subsequent analysis will show that only one alternative seems to be viable if one takes proton–lifetime bounds and the order of magnitude of the weak scale into consideration. It is particularly interesting, therefore, to note that under certain assumptions, current phenomenological input is able to constrain the number of different group-breaking channels, even when these appear at scales much higher than the ones directly accessible at present.

In particular, the Pati–Salam group is taken to break at the scale Λ_{PS} either along the channel

$$SU(4)_{\mathrm{PS}} \times SU(2)_{\mathrm{R}} \longrightarrow SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{Y}}, \tag{4}$$

or along the channel

$$SU(4)_{\mathrm{PS}} \times U(1)_{\mathrm{R}} \longrightarrow SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{Y}} \tag{5}$$

if the breaking $SU(2)_{\mathrm{R}} \longrightarrow U(1)_{\mathrm{R}}$ has already occurred at Λ_{GUT}. A third possibility is also examined, namely one of a Pati–Salam symmetry breaking such as

$$SU(4)_{\mathrm{PS}} \longrightarrow SU(3)_{\mathrm{C}} \times U(1)_{B-L} \tag{6}$$

at Λ_{GUT}, which is followed by the breaking of $SU(2)_{\mathrm{R}} \times U(1)_{B-L} \longrightarrow U(1)_{\mathrm{Y}}$ at scale Λ_{R}. In all these alternative scenarios, the Pati–Salam 4-plets are each broken into a QCD triplet and a lepton, while simultaneously giving rise to a "predecessor" of the electromagnetic charge.

It is also noted that, at a first approximation, below the mirror-mass threshold scale Λ_{M} the couplings of all the nonabelian groups, except for the generation group are taken to evolve with $N_{\mathrm{f}} = 6$ as in the standard model, and to be above Λ_{M} with $N_{\mathrm{f}} = 12$, the doubling being caused by the existence of mirror fermions, which leads to an abrupt change in slope to the running of the couplings at that scale. The eventual top-quark decoupling, as well as the mixing between ordinary and mirror fermions, which apart from the top quark is quite small, is thus also neglected. The $SU(3)_{2\mathrm{G}}$ coupling evolves at all scales with $N_{\mathrm{f}} = 8$. These N_{f} values are the same for all three Pati–Salam- breaking channels considered.

It is more convenient to work in the following with the inverse structure constants $\alpha^{-1} = 4\pi/g^2$, since their evolution is linear with $\ln(p/p_0)$. The value of the hypercharge coupling $\alpha_{\mathrm{Y}}(\Lambda_{\mathrm{PS}})$ in the first two cases is computed via the relation

$$\alpha_{\mathrm{Y}}^{-1} = (3\alpha_{\mathrm{R}}^{-1} + 2\alpha_{\mathrm{PS}}^{-1})/5 \tag{7}$$

which is evaluated at the Pati–Salam scale Λ_{PS}, where α_{R} is the coupling corresponding to $SU(2)_{\mathrm{R}}$ or $U(1)_{\mathrm{R}}$, respectively. In the third case, the hypercharge coupling is given by the relation

$$\alpha_{\mathrm{Y}}^{-1}(\Lambda_{\mathrm{R}}) = (3\alpha_{\mathrm{R}}^{-1}(\Lambda_{\mathrm{R}}) + 2\alpha_{B-L}^{-1}(\Lambda_{\mathrm{R}}))/5. \tag{8}$$

Furthermore, the first and third cases are based on the working assumption of unbroken discrete left–right symmetry above the scale where $SU(2)_{\mathrm{R}}$ is broken, i.e., $\alpha_{\mathrm{R}} = \alpha_{\mathrm{L}}$. As was said in the introduction, a discussion on the possible breaking mechanisms of these symmetries is here avoided, since the purpose of the analysis is to allow general qualitative conclusions to be drawn.

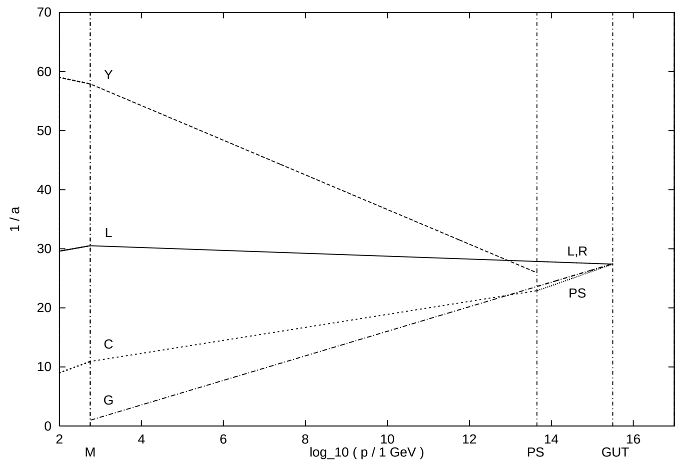

Fig. 1. The running of the inverse fine-structure constants $\alpha_{\mathrm{Y,L,C,G}}^{-1}$ and later $\alpha_{\mathrm{R,PS}}^{-1}$, corresponding to the breaking channel $SU(4)_{\mathrm{PS}} \times SU(2)_{\mathrm{R}} \longrightarrow SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{Y}}$. The vertical lines, starting from small energies, correspond to the scales Λ_{M}, Λ_{PS}, and Λ_{GUT}. The relevant scales are found to be $\Lambda_{\mathrm{M}} = 10^{2.75}$ GeV, $\Lambda_{\mathrm{PS}} = 10^{13.65}$ GeV, and $\Lambda_{\mathrm{GUT}} = 10^{15.5}$ GeV

The starting point of the calculation is based on the approximate experimental values for these quantities listed below [5]

$$\begin{aligned} \alpha_{\mathrm{Y}}^{-1}(M_{\mathrm{Z}}) &\sim 59.2, \\ \alpha_{\mathrm{L}}^{-1}(M_{\mathrm{Z}}) &\sim 29.6, \\ \alpha_{\mathrm{C}}^{-1}(M_{\mathrm{Z}}) &\sim 8.4. \end{aligned} \tag{9}$$

Moreover, at scale Λ_{M}, the $SU(3)_{2\mathrm{G}}$ coupling is taken to be equal to $\alpha_{2\mathrm{G}}(\Lambda_{\mathrm{M}}) = 1$. This coupling is not plotted for $\alpha_{2\mathrm{G}} > 1$, because then higher-order corrections to the renormalization of this coupling become important. This is expected to have a limited effect on the other couplings, however, since the generation group breaks at around the same scale. This issue is examined again later.

The evolution of the inverse fine-structure constants α_i^{-1} for the various couplings $i = \mathrm{Y, L, C, G, PS, R}$ of the groups $U(1)_{\mathrm{Y}}$, $SU(2)_{\mathrm{L}}$, $SU(3)_{\mathrm{C}}$, $SU(3)_{2\mathrm{G}}$, $SU(4)_{\mathrm{PS}}$, and $SU(2)_{\mathrm{R}}$ corresponding to the three different breaking channels mentioned above are plotted consecutively in Figs. 1, 2, and 3. The relevant scales for which unification is possible, along with the value of the unification coupling, are also given in Table 1.

To begin with, we discuss the first two cases. The energy scales of interest are found to be $\Lambda_{\mathrm{M}} = 10^{2.75}$ GeV and $\Lambda_{\mathrm{PS}} = 10^{13.65}$ GeV in the first case, and $\Lambda_{\mathrm{M}} = 10^{2.4}$ GeV and $\Lambda_{\mathrm{PS}} = 10^{12.5}$ GeV in the second. The corresponding unification scales and couplings are found to be $\Lambda_{\mathrm{GUT}} = 10^{15.5}$ GeV and $\alpha_{\mathrm{GUT}} = 0.036$ in the first case, and $\Lambda_{\mathrm{GUT}} = 10^{14.9}$ GeV and $\alpha_{\mathrm{GUT}} = 0.037$ in the second. One can see at the scale Λ_{PS} in Figs. 1 and 2 the characteristic change in slope of the Pati–Salam coupling when the group $SU(4)_{\mathrm{PS}}$ breaks down to $SU(3)_{\mathrm{C}}$; this is due to the different quadratic Casimirs of their adjoint representations as well as the starting of the hypercharge-

Table 1. The energy scales required to achieve unification assuming three different symmetry-breaking channels, and the corresponding value of the unification coupling. The second channel is disfavored because of the low unification scale, and the third channel is also disfavored because of the large mirror-fermion masses implied by $\Lambda_{\rm M}$

Symmetry-breaking sequence assuming an $SU(3)_{\rm 2G}$ generation group	Energy scales (GeV) $\Lambda_{\rm M}$	$\Lambda_{\rm R}$	$\Lambda_{\rm PS}$	$\Lambda_{\rm GUT}$	$\alpha_{\rm GUT}$
$SU(4)_{\rm PS} \times SU(2)_{\rm R} \to SU(3)_{\rm C} \times U(1)_{\rm Y}$	$10^{2.75}$	$\Lambda_{\rm PS}$	$10^{13.65}$	$10^{15.5}$	0.036
$SU(4)_{\rm PS} \times U(1)_{\rm R} \to SU(3)_{\rm C} \times U(1)_{\rm Y}$	$10^{2.4}$	$\Lambda_{\rm PS}$	$10^{12.5}$	$10^{14.9}$	0.037
$SU(4)_{\rm PS} \to SU(3)_{\rm C} \times U(1)_{B-L}$	10^{4}	$10^{10.05}$	$\Lambda_{\rm GUT}$	$10^{17.3}$	0.034

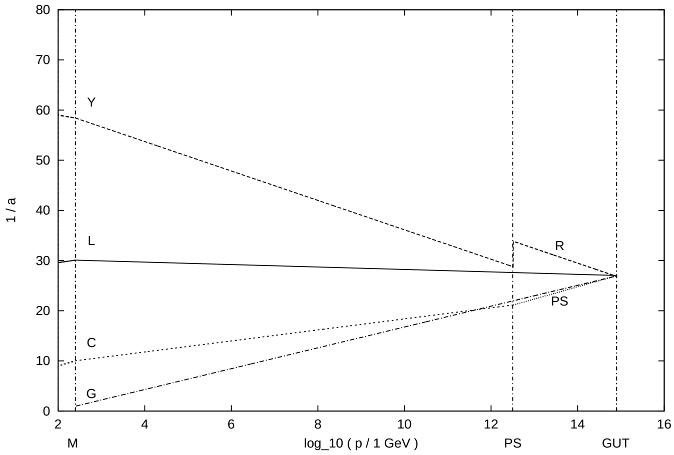

Fig. 2. The running of the inverse fine-structure constants $\alpha^{-1}_{\rm Y,L,C,G}$ and later $\alpha^{-1}_{\rm R,PS}$, corresponding to the breaking channel $SU(2)_{\rm R} \longrightarrow U(1)_{\rm R}$ at $\Lambda_{\rm GUT}$ and $SU(4)_{\rm PS} \times U(1)_{\rm R} \longrightarrow SU(3)_{\rm C} \times U(1)_{\rm Y}$ at $\Lambda_{\rm PS}$. The scales are $\Lambda_{\rm M} = 10^{2.4}$ GeV, $\Lambda_{\rm PS} = 10^{12.5}$ GeV, and $\Lambda_{\rm GUT} = 10^{14.9}$ GeV

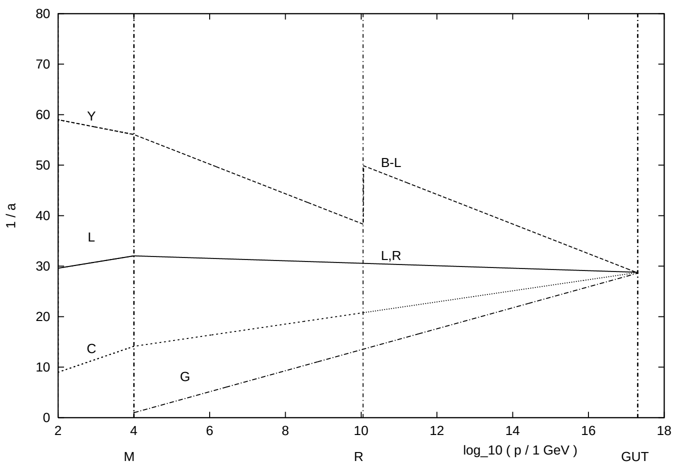

Fig. 3. The running of the inverse fine-structure constants $\alpha^{-1}_{\rm Y,L,C,G}$ and later $\alpha^{-1}_{B-L,\rm R}$, assuming that there is a symmetry-breaking channel such as $SU(4)_{\rm PS} \longrightarrow SU(3)_{\rm C} \times U(1)_{B-L}$ at $\Lambda_{\rm GUT} = 10^{17.3}$ GeV and $SU(2)_{\rm R} \times U(1)_{B-L} \longrightarrow U(1)_{\rm Y}$ at $\Lambda_{\rm R} = 10^{10.05}$ GeV. The vertical lines, starting from small energies, correspond here to the scales $\Lambda_{\rm M}$, $\Lambda_{\rm R}$, and $\Lambda_{\rm GUT}$. The mirror-fermion masses are taken to be $\Lambda_{\rm M} = 10^4$ GeV so that the generation coupling can meet the rest of the gauge couplings

coupling running at that scale, since at scales higher than $\Lambda_{\rm PS}$, $U(1)_{\rm Y}$ is embedded in other groups.

The inclusion of a minimal Higgs field able to break these symmetries spontaneously would, for the same $\Lambda_{\rm GUT}$, slightly shift $\Lambda_{\rm PS}$ downwards, since it would slow down somewhat the running of the Pati–Salam coupling. It is clear from the figures that, with the present fermion content, the slopes of the gauge couplings do not favor $SU(5)$ unification. Also, the slope of $SU(2)_{\rm L}$ below $\Lambda_{\rm M}$ speaks against the addition of new weak-singlet fermions, as is usually done in universal seesaw models [6], if one seeks coupling unification.

The third alternative breaking sequence is, as has already been said, to break the Pati–Salam group at a unification group such as $SU(4)_{\rm PS} \longrightarrow SU(3)_{\rm C} \times U(1)_{B-L}$, and have later the breaking $SU(2)_{\rm R} \times U(1)_{\rm R} \longrightarrow U(1)_{\rm Y}$ at scale $\Lambda_{\rm R}$. This possibility is drawn in Fig. 3. The relevant scales are found to be $\Lambda_{\rm M} = 10^4$ GeV, $\Lambda_{\rm R} = 10^{10.05}$ GeV and $\Lambda_{\rm GUT} = 10^{17.3}$ GeV. The scale $\Lambda_{\rm R}$ with the present fermion content is quite large (cf. solutions with alternative fermion contents, for example [7]), supporting a seesaw mechanism for the neutrino masses. The unification scale is in this case quite large, a result reminiscent of [8], and the common coupling at that scale is $\alpha_{\rm GUT} = 0.034$. The main reason for the largeness of $\Lambda_{\rm M}$ is the effort to unify the generation-group gauge coupling with the other couplings.

If this unification condition is relaxed in the same way it is relaxed in connection with an $SU(4)_{\rm 2G}$ generation group that is discussed in the next subsection, the rest of the couplings can be unified with a smaller $\Lambda_{\rm M}$ and this channel is still viable. Nonetheless, in the present case, the largeness of $\Lambda_{\rm M}$ used would correspond to an unacceptably large weak scale. The fact that $SU(2)_{\rm R}$ breaks far away from $\Lambda_{\rm GUT}$ would also render effects coming from the mechanism to which its breaking is due (and which are here neglected, like the existence of scalar particles) more important. Such effects, however, are not expected to alleviate the problem of the large scale $\Lambda_{\rm M}$. We can therefore conclude here that the third breaking channel is improbable, unless generation-coupling unification with the rest of the couplings is abandoned. Another way to keep $\Lambda_{\rm M}$ small would be of course to add a large Higgs sector transforming nontrivially under $SU(3)_{\rm 2G}$, but this alternative is not investigated, since it is foreign to the

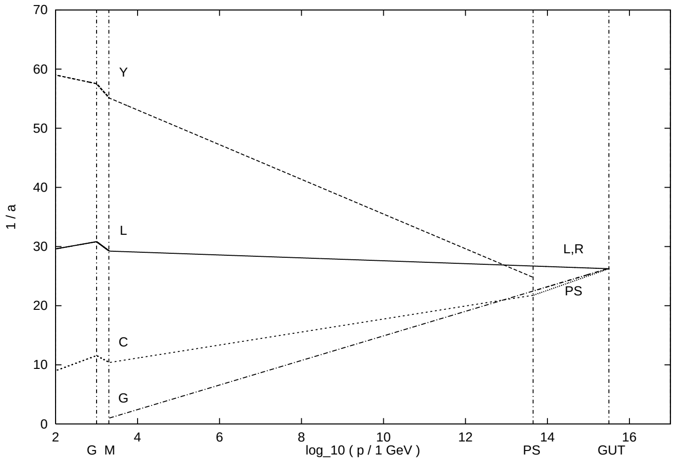

Fig. 4. The running of the inverse fine-structure constants $\alpha^{-1}_{\mathrm{Y,L,C,G}}$ and later $\alpha^{-1}_{\mathrm{R,PS}}$, corresponding to the breaking channel $SU(4)_{\mathrm{PS}} \times SU(2)_{\mathrm{R}} \longrightarrow SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{Y}}$, to simulate a possible large influence of the strong generation group $SU(3)_{2\mathrm{G}}$. It is found that in order to achieve unification, one needs $\Lambda_{\mathrm{G}} = 10^3\,\mathrm{GeV}$, $\Lambda_{\mathrm{M}} = 10^{3.3}\,\mathrm{GeV}$, $\Lambda_{\mathrm{PS}} = 10^{13.65}\,\mathrm{GeV}$, and $\Lambda_{\mathrm{GUT}} = 10^{15.5}\,\mathrm{GeV}$

present conceptual framework and would raise naturalness problems.

Gauge-coupling unification in connection with bounds on the proton lifetime is discussed next, since the breaking of $SO(10)_{\mathrm{D}}$ at Λ_{GUT} can induce proton decay via effective four-fermion operators. This issue could actually help us decide between the first two breaking channels proposed. From the proton-lifetime experimental constraint [5]

$$\tau(\mathrm{p} \to \mathrm{e}^+\pi^0) > 5.5 \times 10^{32}\ \mathrm{yr} \tag{10}$$

and the theoretical order-of-magnitude estimate

$$\tau^{-1} \approx \alpha^2_{\mathrm{GUT}} \frac{m^5_{\mathrm{p}}}{\Lambda^4_{\mathrm{GUT}}}, \tag{11}$$

one gets the inequality

$$\alpha_{\mathrm{GUT}} < 0.074 \left(\frac{\Lambda_{\mathrm{GUT}}}{10^{15.5}}\right)^2. \tag{12}$$

This proton-lifetime bound makes clear that the second breaking channel possibility is disfavored because of a small unification scale. Nevertheless, it cannot be at this point definitely excluded, because there is a limited level of accuracy of the current rather qualitative analysis. Note that this result is reminiscent of the result of [8] in an analysis with the same breaking sequence in a left–right-symmetric context but without mirror fermions. One is consequently left with the first alternative as the one corresponding to the most probable symmetry-breaking sequence.

2.3 The effect of strong dynamics

Strong dynamics can alter the results quoted above, since higher-order corrections to the various β functions due to the strong $SU(3)_{2\mathrm{G}}$ interactions could become important if the quantity r introduced before is not negligible. However, as is also noted in [9], the fermion content of the theory implies that this effect, however large, would be uniform for all standard-model couplings, as is shown in Fig. 4 for an exaggerated effect corresponding to $r = 40$. From (3), this would correspond to a highly nonperturbative generation coupling $\alpha_{\mathrm{G}} \approx 126$ (this number is of course purely indicative, since the perturbative β function has no meaning in this regime), something which has the same influence on each of the other relatively weak $SU(N)$ couplings as the introduction of 20 new fermion N-plets.

Such strong dynamics can shift the unification coupling α_{GUT} to larger values, but cannot shift the unification scale. In reality, the coupling-evolution curves shown should be smooth, without angles, but r is here taken to become suddenly important for illustration purposes, in a perhaps overambitious effort to simulate the relevant effect. There is no guarantee, of course, that the nonperturbative effects of $SU(3)_{2\mathrm{G}}$ can be limited even by such large r values, but to maintain the conclusions presented in this work, it is assumed that they are.

The analysis of this alternative leads one to split the scale where the mirror fermions decouple Λ_{M} from the scale Λ_{G} where the generation group becomes strong, and to consider, for instance, the most probable breaking channel corresponding to Fig. 1. An effort is therefore made to "parametrize" by means of the quantity r our ignorance of the strong dynamics and the effects they have on the other couplings in the energy region between the scales Λ_{G} and Λ_{M}. The unification and Pati–Salam scale remain the same as in Fig. 1, but the scale Λ_{M} has to be raised to $10^{3.3}\,\mathrm{GeV}$, and one has to have a further a new scale $\Lambda_{\mathrm{G}} = 10^3\,\mathrm{GeV}$ in order to achieve unification. The strong coupling tends to make the other couplings slightly larger at Λ_{GUT}, i.e., one gets $\alpha_{\mathrm{GUT}} = 0.037$. If these effects are really large, difficulties with the reproduction of the weak scale could potentially arise from the heaviness of the mirror fermions.

2.4 Mirror generation groups other than $SU(3)_{2\mathrm{G}}$

The issue of the generation groups comes next. Even if there is an abelian generation group $U(1)_{\mathrm{G}}$ surviving down to TeV scales along with $SU(3)_{2\mathrm{G}}$, unification requires that its coupling be negligibly small at low energies, so its running is neglected and its evolution with energy not plotted. Note, however, that if one wants the seesaw mechanism to work for the standard-model neutrinos, as will be seen in the next section, this symmetry would have to be broken at very large energy scales to allow for ultralight neutrinos. In the first case considered, for instance, one could think of a breaking channel involving $U(1)_{\mathrm{G}}$, like $SU(4)_{\mathrm{PS}} \times SU(2)_{\mathrm{R}} \times U(1)_{\mathrm{G}} \longrightarrow SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{Y}} \times U(1)_{G'}$, with the standard-model neutrinos being $U(1)_{G'}$-neutral. This does not change the previous results and conclusions, but it could alter the values of the abelian generation-group charges in [1].

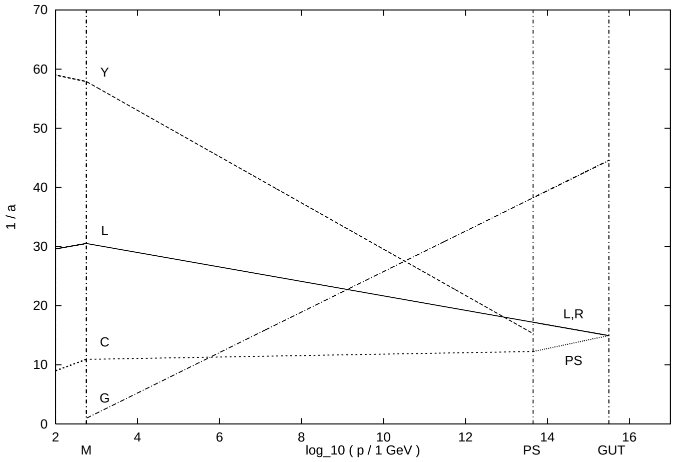

Fig. 5. The running of the inverse fine-structure constants $\alpha^{-1}_{\mathrm{Y,L,C,G}}$ and later $\alpha^{-1}_{\mathrm{R,PS}}$, corresponding to the breaking channel $SU(4)_{\mathrm{PS}} \times SU(2)_{\mathrm{R}} \longrightarrow SU(3)_{\mathrm{C}} \times U(1)_{\mathrm{Y}}$. The generation group $SU(4)_{2\mathrm{G}}$ is here taken to be unbroken until TeV scales, and unification of its coupling with the rest of the gauge couplings is here abandoned. All couplings apart from the generation coupling are larger at the unification scale due to the existence of two additional generations of fermions. The relevant scales are found to be, as in Fig. 1, equal to $\Lambda_{\mathrm{M}} = 10^{2.75}$ GeV, $\Lambda_{\mathrm{PS}} = 10^{13.65}$ GeV, and $\Lambda_{\mathrm{GUT}} = 10^{15.5}$ GeV

One could also give up unification of the generation-group couplings with the other couplings. This would mean either letting the relation between g_1 and g_2 free, or considering $SU(4)_{2\mathrm{G}}$ instead of $SU(3)_{2\mathrm{G}}$ unbroken down to low-energy scales. The latter would correspond to having also a fourth fermion generation paired up with its mirror partner at scales of the order of 1 TeV; this would make $N_{\mathrm{f}} = 16$ for the standard-model groups instead of $N_{\mathrm{f}} = 12$ used in all previous cases. The generation-group β function would remain with $N_{\mathrm{f}} = 8$ as before. The corresponding running of the couplings is plotted in Fig. 5. The scales Λ_{M}, Λ_{PS}, and Λ_{GUT} remain the same as in the more favored case of Fig. 1.

This scenario has the advantage that it can generate lepton masses through a strong $U(1)_{\mathrm{G}}$ coupling after its breaking at around 1 TeV [1]. It suffers, however, from the same problem as the one encountered in [2], since in both cases the generation coupling is running too fast and complete unification is lost. One could achieve unification only by pushing the scale Λ_{M} to very high values and thus paying the unacceptable price of a weak scale several orders of magnitude larger than what it should be. Moreover, because of the introduction of additional low-lying fermions, it pushes the other gauge couplings to larger values at the unification point, since one finds $\alpha_{\mathrm{GUT}} = 0.067$, something which could potentially create problems with proton decay. This is also the reason why no additional fermion generations or their mirrors should be generally expected much below the unification scale.

Important conclusions can be therefore drawn here, namely that for unification of the generation coupling with the other gauge couplings, one should have a group $SU(3)_{2\mathrm{G}}$ becoming strong at around 1 TeV whose coupling renormalization naturally reproduces the hierarchy between the QCD and the weak scale, and which argues against alternative generation groups unbroken to low energies like $SU(4)_{2\mathrm{G}}$ or $SU(2)_{2\mathrm{G}}$ for instance. This is, to the best of our knowledge, the first example of a fully unifiable and phenomenologically viable dynamical symmetry-breaking model.

Note, moreover, that in all cases considered, the Pati–Salam group has to break at a very high energy to achieve unification, which is due to the fast running of the Pati–Salam coupling, making it very difficult to feed down quark masses to leptons. In order to avoid light fundamental Higgses therefore, the existence of gauge-invariant operators of the form $\bar{l}_{\mathrm{R}} l^{\mathrm{M}}_{\mathrm{L}} \bar{q}^{\mathrm{M}}_{\mathrm{R}} q_{\mathrm{L}}$ which arise nonperturbatively would have to be postulated as generating lepton masses.

3 Neutrino masses and mixings

3.1 The structure of the mass matrix

Having acquired a general idea on how and at what energy scales the gauge symmetries break in this model, we present an investigation of the neutrino mass hierarchy and mixing angles next. Since the symmetry-breaking channels considered involve always the breaking of the $SU(2)_{\mathrm{R}}$ and $SU(3)_{2\mathrm{G}}$ symmetries, which protect neutrino and mirror neutrino Majorana masses respectively, it is most natural to ask now what happens to their relevant masses and mixings after these breakings. In light of recent experimental results on neutrinos [10], such considerations go beyond a mere academic interest.

Even though the analysis that follows is not exhaustive and the mass-matrix entries do not stem from a specific calculational scheme, this example not only demonstrates explicitly the power that the present framework has to describe several phenomenological issues without fine-tuning of parameters, but it also leads to quite useful general conclusions about the structure of the mirror-lepton submatrices. For simplicity, the lepton mass matrices, as well as their submatrices $M_{\mathrm{L}}, m_{\mathrm{L}}, M, \tilde{M}, m_{\mathrm{I}}, m_{\mathrm{B}}$, defined below, are taken as symmetric and real, ignoring CP-violating phases which may arise in the lepton sector.

Recalling the numerical example in [1], for the charged leptons l, a 6×6 mass matrix $\mathcal{M}_{\mathrm{L}}$ is introduced, having the form

$$
\begin{array}{cc}
 & \begin{array}{cc} l_{\mathrm{L}} & l^{\mathrm{M}}_{\mathrm{L}} \end{array} \\
\begin{array}{c} \bar{l}_{\mathrm{R}} \\ \bar{l}^{\mathrm{M}}_{\mathrm{R}} \end{array} & \begin{pmatrix} 0 & m_{\mathrm{L}} \\ m_{\mathrm{L}} & M_{\mathrm{L}} \end{pmatrix}
\end{array} \qquad (13)
$$

with the superscripts M indicating mirror fields and M_{L}, m_{L} being 3×3 matrices in a generation space given by

$$
M_{\mathrm{L}}(\mathrm{GeV}) = \begin{pmatrix} 180 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 200 \end{pmatrix},
$$

$$m_{\mathrm{L}}(\mathrm{GeV}) = \begin{pmatrix} 0.25 & 0.25 & 0.1 \\ 0.25 & 3.8 & 1 \\ 0.1 & 1 & 17 \end{pmatrix}. \tag{14}$$

These produce, after diagonalization, the following lepton and mirror-lepton mass hierarchy (at 2 TeV and in GeV units):

Standard-model charged leptons Mirror charged leptons $m_\tau = 1.45$, $m_\mu = 0.07$, $m_{\mathrm{e}} = 3 \times 10^{-4}$ $m_{\tau^{\mathrm{M}}} = 201$, $m_{\mu^{\mathrm{M}}} = 200$, $m_{\mathrm{e}^{\mathrm{M}}} = 180$.

Recall that the order of magnitude of these masses has to fulfill the double requirement of escaping direct detection in current high-energy facilities and reproducing along with the other mirror fermions the correct weak scale.

For the neutrinos the situation is more complicated, since they could be of a Majorana nature. The neutrino 12×12 mass matrix $\mathcal{M}_{\mathrm{N}}$ is introduced next, and it is taken to have the following form:

$$\begin{array}{c|c} & \nu_{\mathrm{L}} \quad \nu_{\mathrm{R}}^{c} \quad \nu_{\mathrm{L}}^{\mathrm{M}} \quad \nu_{\mathrm{R}}^{\mathrm{M},c} \\ \begin{array}{c} \bar{\nu}_{\mathrm{L}}^{c} \\ \bar{\nu}_{\mathrm{R}} \\ \bar{\nu}_{\mathrm{L}}^{\mathrm{M},c} \\ \bar{\nu}_{\mathrm{R}}^{\mathrm{M}} \end{array} & \begin{pmatrix} 0 & 0 & 0 & m_{\mathrm{I}} \\ 0 & M & m_{\mathrm{I}} & 0 \\ 0 & m_{\mathrm{I}} & \tilde{M} & m_{\mathrm{B}} \\ m_{\mathrm{I}} & 0 & m_{\mathrm{B}} & 0 \end{pmatrix} \end{array}, \tag{15}$$

where the entries shown are 3×3 matrices in generation space. The zero blocks are protected by the $SU(2)_{\mathrm{L}}$ symmetry. The matrices m_{B} and m_{I} denote Dirac mass matrices having $SU(2)_{\mathrm{L}}$-breaking and $SU(2)_{\mathrm{L}}$-invariant entries, respectively, and $\tilde{M}, M$ are Majorana mass matrices with $SU(2)_{\mathrm{R}} \times U(1)_{\mathrm{G}}$-breaking elements for the mirror and ordinary neutrinos, respectively. The structure of these matrices in generation space determines the mass hierarchies and mixings of the neutrinos.

Since the present model does not predict the existence of a light sterile neutrino, contrary to other "mirror" models [11], attention is restricted to current experimental data regarding solar and atmospheric neutrino anomalies, which imply differences of masses squared $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, with $i,j = 1,2,3$, and mixing angles θ among only three mass eigenstates $m_{1,2,3}$ of standard-model left-handed neutrinos, if one accepts the view that they are due to quantum-mechanically coherent oscillations between different neutrino-flavor eigenstates. Assuming, for instance, that the small-angle MSW solution to the solar-neutrino deficit involves the left-handed electron and muon standard-model neutrinos ν_{e} and ν_μ, one gets experimental bounds which according to [10] are given by

$$4 \times 10^{-6}\mathrm{eV}^2 \lesssim \Delta m_{21}^2 \lesssim 1.2 \times 10^{-5}\mathrm{eV}^2 \tag{16}$$

with a mixing angle $\sin\theta_{\mathrm{sun}} \approx 0.03 - 0.05$.

Further information on neutrino masses and mixing coming from the atmospheric neutrino anomaly, assuming it involves the left-handed standard-model neutrinos ν_τ and ν_μ, gives the bounds

$$4 \times 10^{-4}\mathrm{eV}^2 \lesssim \Delta m_{31}^2 \lesssim 8 \times 10^{-3}\mathrm{eV}^2 \tag{17}$$

with a mixing angle $\sin\theta_{\mathrm{atm}} \approx 0.49 - 0.71$ associated with it [10]. It has to be noted that this $\nu_\mu - \nu_\tau$ mixing is unusually large as compared with charged-fermion mixings observed so far. This experimental input constitutes the basis which determines the form of the neutrino submatrices given below.

3.2 Two numerical examples of mass matrices

The working assumption is made next that the $SU(2)_{\mathrm{L}}$-breaking mirror-fermion Dirac masses satisfy the inequalities $m_{U^{\mathrm{M}}} > m_{\nu^{\mathrm{M}}} > m_{l^{\mathrm{M}}}$ for each fermion generation, where U^{M} stands for an up-type mirror-quark field. This is done in analogy with the standard model, where each generation contains quarks that are heavier than the corresponding leptons. Up-type quarks are heavier than down-type quarks only for the two heavier standard-model generations, but it is imagined in the present example that this is a general property for mirror quarks and leptons in all their generations.

Taking into consideration the charged-fermion mass matrices [1] and the $SU(2)_{\mathrm{R}}$ symmetry-breaking scale, the phenomenological input given above leads to the following choice of mass matrices:

$$m_{\mathrm{B}}(\mathrm{GeV}) = \begin{pmatrix} 250 & 0 & 0 \\ 0 & 350 & 0 \\ 0 & 0 & 350 \end{pmatrix},$$

$$m_{\mathrm{I}}(\mathrm{GeV}) = \begin{pmatrix} 20 & 1 & 1 \\ 1 & 50 & 20 \\ 1 & 20 & 70 \end{pmatrix} \tag{18}$$

and

$$\tilde{M} = 0, \ M = 10^{13}\, I_3 \quad \mathrm{GeV}, \tag{19}$$

with the m_{B} entries being generated as in the matrix M_{L} by the strong $SU(3)_{2\mathrm{G}}$ interaction which breaks $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ dynamically [1], and I_3 the 3×3 identity matrix. Dirac mirror neutrinos are initially considered, since $\tilde{M} = 0$. It will become clear in the following that the magnitude of the m_{I} entries, in conjunction with M, seems to be crucial for the reproduction of the correct mixings of (16) and (17). Once the magnitude of the M entries is fixed, the structure of m_{I} is therefore more or less constrained.

In the next section, Majorana mirror neutrinos are also studied, which leads to the introduction of $\tilde{M} \neq 0$, but it is noted that the Dirac or Majorana nature of the mirror neutrinos does not influence substantially the masses and mixings of the ordinary neutrinos for which one has experimental evidence to compare with the relative theoretical predictions. Therefore, Dirac mirror neutrinos are suitable for the purposes of this section. The scale of the large Majorana masses is taken to be close to the scale where the $SU(2)_{\mathrm{R}}$ symmetry is broken. (Gauge invariance dictates, of course, that this breaking is due to a nonzero vacuum expectation value of an $SU(2)_{\mathrm{R}}$ triplet.) The matrix M is chosen as diagonal for simplicity, even though in principle the large $\nu_\mu - \nu_\tau$ mixing could originate also from this matrix.

Moreover, the mirror neutrino matrices $m_{\rm B}, \tilde{M}$ are also chosen as diagonal for simplicity, since mirror mixing can be fed down only indirectly to ordinary fermions and can hardly account for the large observed $\nu_\mu - \nu_\tau$ neutrino mixing. Therefore, even if mirror-neutrino mixing is present, lack of relevant experimental evidence would just burden the numerical example with more parameters, so it is ignored. What is more, large nondiagonal elements in $m_{\rm B}$ would lead to dangerously light weak-doublet mirror neutrinos. One is therefore left to try large nondiagonal elements for the matrix $m_{\rm I}$ in order to explain at least part of the large neutrino mixing.

The numerical example above leads after diagonalization of the mass matrix to the following neutrino mass hierarchy (at 2 TeV):

Standard-model (Majorana) neutrinos Mirror (Dirac) neutrinos (GeV) $m_{\nu_{3\rm L}} = 0.03$ eV, $m_{\nu_{2\rm L}} = 0.002$ eV, $m_{\nu_{1\rm L}} = 0.00003$ eV $m_{\nu_{3\rm M}} = 201$, $m_{\nu_{2\rm M}} = 200$, $m_{\nu_{3\rm R}} \approx m_{\nu_{2\rm R}} \approx m_{\nu_{1\rm R}} = 10^{13}\,\text{GeV}$ $m_{\nu_{1\rm M}} = 180$.

The mass matrix $\mathcal{M}_{\rm N}$ gives also a mixing $\sin\theta$ for the solar and atmospheric neutrino problems (which in our case of course involve only standard-model neutrinos). equal to 0.04 and 0.53, respectively, which is compatible with experiment [10].

Whereas the form of $m_{\rm B}$ is consistent with the corresponding mirror charged-lepton matrix $M_{\rm L}$, the matrix $m_{\rm I}$ is slightly problematic, since it has a gauge-invariant mass term $\bar{\nu}_{\mu\,L}\nu^{\rm M}_{\mu\,R}$ for the second generations which is larger than the corresponding $\bar{c}_{\rm L}c^{\rm M}_{\rm R}$ quark-mass term in [1]. Qualitatively, one would generally expect such terms involving quarks to be larger than the corresponding lepton ones. Although this is generally expected in analogy with the standard-model case and possibly based on QCD-related contributions to particle masses, the lack of a definite calculational scheme for these gauge-invariant masses poses limits to such arguments. There are nevertheless several solutions to this naturalness issue, and these are presented below.

One possibility is to consider lighter mirror leptons, which would then allow for smaller entries in $m_{\rm I}$. This by itself would, however, not be enough to remove this discrepancy without exceeding the lower mass bounds on direct production of new weak-doublet fermions set by the LEP experiments. Another solution is to consider heavier mirror quarks and charged leptons, which would then require larger entries in the corresponding gauge-invariant quark-mass submatrices [1]. This solution would also help indirectly to reduce the problems with the electroweak precision tests, as is shown in the next section.

A third alternative solution to this naturalness question is here investigated, i.e., smaller Majorana masses for the fields $\nu_{\rm R}$. This is not such a severe assumption, since in nature, one has already examples like the electron, which has a mass more than five orders of magnitude smaller than the scale where the symmetry which forbids its mass breaks. The choice $M = 10^{11} I_3\,\text{GeV}$ is made next, and $m_{\rm B}$ is kept the same as before, in which case the matrix $m_{\rm I}$ takes the form

$$m_{\rm I}(\text{GeV}) = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 17 & 5 \\ 0 & 5 & 21 \end{pmatrix}. \tag{20}$$

As in the previous case, the closeness of the (2,2) and (3,3) entries is crucial if one wants to generate large $\nu_\mu - \nu_\tau$ mixing without nondiagonal (2,3) and (3,2) entries that would be inconsistently large in comparison with the corresponding entries of the other fermion mass matrices. Even though nondiagonal entries could also reproduce the correct neutrino mixing by being smaller than the ones chosen here, provided the diagonal entries were even closer to each other, something which would be reminiscent of the maximally mixed $K^0 - \bar{K}^0$ system, such a scenario would fail to produce the required mass hierarchies. Using the mass matrix in (20) gives rise to the following neutrino mass hierarchy (at 2 TeV):

Standard-model (Majorana) neutrinos Mirror (Dirac) neutrinos (GeV) $m_{\nu_{3\rm L}} = 0.05$ eV, $m_{\nu_{2\rm L}} = 0.002$ eV, $m_{\nu_{1\rm L}} = 0.00004$ eV $m_{\nu_{3\rm M}} = 201$, $m_{\nu_{2\rm M}} = 200$, $m_{\nu_{3\rm R}} \approx m_{\nu_{2\rm R}} \approx m_{\nu_{1\rm R}} = 10^{11}\,\text{GeV}$ $m_{\nu_{1\rm M}} = 180$ and mixing angles similar to the ones in the previous case.

Even smaller Majorana masses would potentially lead to neutrino masses on the order of 1 eV, which could be of cosmological interest because of hot dark matter. However, having Majorana neutrino masses even lighter than two orders of magnitude smaller than the $SU(2)_{\rm R}$-breaking scale seems unlikely and such a possibility is not studied. A similar possibility would be to have a contribution on the order of 1 eV to the Dirac sector of all standard-model neutrinos due to unspecified effects, but since the present model cannot calculate or predict such effects, this issue is also not pursued further.

It is worth noting here that, in order to get the observed fermion mass hierarchies and mixings [1], one is led to consider gauge-invariant fermion mass terms $\bar{\psi}^{\rm M}_{\rm L}\psi_{\rm R}$, denoted by $m_{I\,\psi}$ for $\psi = {\rm t, c, b, s}, \tau, \mu, \nu_\tau, \nu_\mu$, exhibiting the hierarchy

$$\frac{m_{I,\rm t}}{m_{I,\rm c}}, \frac{m_{I,\rm b}}{m_{I,\rm s}} \gg \frac{m_{I,\tau}}{m_{I,\mu}} \gg \frac{m_{I,\nu_\tau}}{m_{I,\nu_\mu}}. \tag{21}$$

It therefore seems that the more gauge interactions a fermion flavor has, the larger the gauge-invariant mass splitting between its third- and second-generation representatives, even when these gauge interactions are relatively weak as compared to the $SU(3)_{2\rm G}$ one. This could be an indication of near-critical four-fermion interactions in the sense explained in [12] and which are contained in the scenario presented in [1]. The large neutrino mixing suggested by the superkamiokande data is therefore consistent with the existence of such types of interactions, as made is clear by the generic pattern noted in (21).

3.3 The mixing parameters

The symmetric charged- and neutral-lepton mass matrices used above are diagonalized by the 6×6 and 12×12

matrices, which we denote by K_i, $i = \mathrm{L}, \mathrm{N}$, respectively, via the relations

$$\mathcal{M}_i = K_i J_i K_i^\dagger, \tag{22}$$

where the J_i denote diagonal matrices. The lepton mixing information is therefore contained in a 6×12 matrix U defined by the relation

$$U_{lj} = (K_\mathrm{L}^\dagger)_{lm} (K_\mathrm{N})_{\nu_m j}, \tag{23}$$

where m summation is implied, with $l, m = \mathrm{e}, \mu, \tau, \mathrm{e}^\mathrm{M}, \mu^\mathrm{M}, \tau^\mathrm{M}$, with ν_m running only over the $SU(2)_\mathrm{L}$-doublet neutrino flavors, and $j = 1, ..., 12$. Note that, following the convention of [10] and contrary to the neutrino case, we keep flavor indices for the charged-lepton mass eigenstates due to the assumed small mixing between them.

It is therefore clear that the three ordinary-neutrino and three mirror-neutrino flavor eigenstates ν_l, which are weak doublets, are given in terms of the twelve neutrino mass eigenstates ν_j via the relation $\nu_l = U_{lj}\nu_j$, with j summation implied. The first three neutrino mass eigenstates are light enough to allow their superposition to be considered as coherent. Furthermore, since the matrix M_L is almost diagonal, K_L is close to the unit matrix, and the form of U is mostly affected by K_N.

Experimentally, there is presently information on only some of the elements of a 3×3 submatrix of U involving standard-model left-handed neutrinos and denoted by U_{st}^SM, with $s = \mathrm{e}, \mu, \tau$ and $t = 1, 2, 3$. The above mass matrices allow the calculation of U^SM by means of (22) and (23), and this is found to be equal (in absolute values) to

$$|U^\mathrm{SM}| = \begin{pmatrix} \sim 1 & 0.039 & 0.01 \\ 0.04 & 0.87 & 0.5 \\ 0.008 & 0.5 & 0.86 \end{pmatrix}. \tag{24}$$

Its form is quasi-symmetric, as is expected from the form of the mass matrices assumed. This is consistent with the matrix given in [10] for the small-angle MSW solution to the solar-neutrino deficit, even though in the present case, U^SM is not rigorously unitary because of the existence of mirror leptons, which slightly mix with the ordinary ones. Moreover, it is observed that the smallness of the element $U_{\mathrm{e}3}^\mathrm{SM}$ justifies in the current example the assumption that the two oscillations are practically decoupled [10].

Larger nondiagonal entries in the matrix m_I can further increase the entries (2,3) and (3,2) and the corresponding $\nu_\mu - \nu_\tau$ mixing. A similar analysis therefore could also be easily performed for the large mixing-angle MSW and the vacuum-oscillation solutions for the solar-neutrino problem, without alteration of the conclusions drawn above about the possibility of having heavier mirror fermions than previously imagined for naturalness reasons, since these are based only on the heavier neutrino mass eigenstates.

4 Mirror neutrinos and the S parameter

The contributions of the mirror fermions to the electroweak precision parameters S and T were calculated in [1] with Dirac mirror neutrinos assumed. Since the generation symmetries which prohibit mirror Majorana masses are broken at around 2 TeV, it is natural to consider Dirac-Majorana mirror neutrinos next. This can be achieved by introducing a nonzero matrix $\tilde{M}$ with entries near that scale, for example $\tilde{M} = 600 I_3$ GeV. The standard-model masses and mixings do not change substantially with this introduction, while the Dirac-Majorana mirror neutrino mass hierarchy takes now the form (in GeV):

$m_{\nu_{3\mathrm{R}}^\mathrm{M}} = 169$, $m_{\nu_{2\mathrm{R}}^\mathrm{M}} = 162$, $m_{\nu_{1\mathrm{R}}^\mathrm{M}} = 91$
$m_{\nu_{3\mathrm{L}}^\mathrm{M}} = 768$, $m_{\nu_{2\mathrm{L}}^\mathrm{M}} = 761$, $m_{\nu_{1\mathrm{L}}^\mathrm{M}} = 691$.

From the identification $m_a \equiv m_{\nu_\mathrm{R}^\mathrm{M}}$ and $m_b \equiv m_{\nu_\mathrm{L}^\mathrm{M}}$ for notational convenience for each of the three mirror-neutrino generations, in the limit $m_{a,b,l} \gg m_\mathrm{Z}$ the oblique leptonic contribution to the S parameter for each mirror generation having a charged lepton of mass m_L is given by [13]

$$S_\mathrm{L}^0 = \frac{1}{6\pi}\{c_\theta^2 \ln(m_a^2/m_\mathrm{L}^2) + s_\theta^2 \ln(m_b^2/m_\mathrm{L}^2) + 3/2 - s_\theta^2 c_\theta^2 [8/3 + f_1(m_a, m_b) - f_2(m_a, m_b) \ln(m_a^2/m_b^2)]\}, \tag{25}$$

where

$$f_1(m_a, m_b) = \frac{3m_a m_b^3 + 3m_a^3 m_b - 4m_a^2 m_b^2}{(m_a^2 - m_b^2)^2}$$
$$f_2(m_a, m_b) = \frac{m_a^6 - 3m_a^4 m_b^2 + 6m_a^3 m_b^3 - 3m_a^2 m_b^4 + m_b^6}{(m_a^2 - m_b^2)^3}. \tag{26}$$

This result is identical to the one given in [14] in this mass limit only if the quantities c_θ and s_θ are correctly defined as

$$c_\theta^2 = 1 - s_\theta^2 = m_b/(m_a + m_b). \tag{27}$$

In the above, corrections due to the fact that one mirror neutrino is not much heavier than the Z boson are neglected, since the purpose of this example is just to illustrate an effect that depends only on mass ratios and not on independent masses, and since one has poor knowledge of the overall mirror-fermion mass normalization anyway. Note, moreover, that contrary to [13,14], the mirror neutrino masses $m_{a,b}$ do not correspond to pure weak eigenstates, because of mixing with ordinary neutrinos. This mixing is small, however, because of the relative smallness of the elements of m_I compared to the m_B entries, and its effects are therefore also neglected.

As regards the $\Delta\rho$ parameter, which measures the isospin-breaking in the new sector, it is shown in [1] that there exists no problem in rendering it small enough to fit experiment, even though some fine-tuning might be needed. Since any leptonic contributions due to Majorana mirror neutrinos as described in [13,14] can be compensated by a corresponding shift to the up–down mass splitting of the mirror fermions, there is no use discussing it further in the present context when the precise mirror-fermion mass spectrum remains experimentally unknown.

The total oblique correction S^0 to S in this model, assuming QCD-like dynamics, is the sum of the contributions S^0_q and S^0_l coming from mirror quarks and leptons, respectively, i.e., [1]

$$S^0 = S^0_q + S^0_l = 0.9 + 0.3. \tag{28}$$

For the light mirror charged leptons chosen in the previous section, the change in S^0_l due to the Dirac-Majorana nature of the mirror neutrinos is marginal, i.e., $S^0_l = 0.12$ instead of $S^0_l = 0.3$ for the case of Dirac mirror neutrinos. If one chooses heavier charged mirror leptons, the change in S^0_l is larger. For instance, for $m_{l^{\rm M}} \approx 400\,\mathrm{GeV}$ one gets $S^0_l = -0.12$, and for $m_{l^{\rm M}} \approx 600\,\mathrm{GeV}$ one gets $S^0_l = -0.24$. The fact that negative S values are currently favored by experiment [15] could therefore be an indication that the mirror leptons are heavier than the ones of about 200 GeV chosen in [1]. The lightest mirror neutrino cannot be much lighter than what it is taken as here, because smaller values for its mass are excluded by present experiments.

After analyzing the above formula for S, we conclude that contributions to S_l are not very sensitive to m_b, but depend drastically on m_a/m_l. Heavier mirror leptons would produce an even smaller S parameter, but assuming that the mirror quarks are at least as heavy as they would render difficult the correct reproduction of the weak scale after a certain point. It is nevertheless clear that a larger m_a/m_l hierarchy could facilitate the reproduction of a small or even negative S parameter in accordance with experiment. This could be achieved now with the assistance of vertex corrections and non-QCD-like dynamics in these models as described in [1] without very large top-quark anomalous couplings having to be introduced. Such a situation would also reduce the fine-tuning needed to keep the $\Delta\rho$ parameter small.

5 Conclusions

Mirror fermions near the weak scale offer rich possibilities for the study of new physics. The absence of direct experimental evidence on the existence of mirror partners to the standard-model fermions led to the present qualitative study of various unification possibilities and related neutrino physics, in which there was no a prior knowledge of the exact mirror mass hierarchies. However, this did not prevent very useful general conclusions about such types of models from being drawn. There are two basic results to be kept in mind. One is that unification of all the gauge couplings, including the generation-group coupling, is possible within this group-theoretical context and consistent not only with the weak scale but also with current bounds on the proton lifetime. The other result is that neutrino masses and mixings consistent with the observed solar and atmospheric neutrino anomalies are naturally achieved.

In particular, it is made clear that with the proposed fermion content extension, not only is $SU(5)$ unification is disfavored, but unification with an $SU(3)_{\rm 2G}$ generation group is possible. Within the group-theoretic framework chosen, this is possible if one takes the gauge coupling (g_1) of one sector to be much larger than the other (g_2) at the unification scale. This unification is *a priori* not at all obvious, and constitutes a highly nontrivial result within the context of dynamical symmetry-breaking theories. There exist nevertheless no direct indications that the scales $\Lambda_{\rm M}$ and $\Lambda_{\rm PS}$ assume indeed the exact values needed for this to happen, and no guarantee that this coupling crossing is not just a coincidence with no particular importance for the embedding of the standard-model gauge structure.

Moreover, the existence of mirror fermions that are weak singlets is also not favored. This is a clear manifestation of the "$\sin\theta_W$" problem in [6,16] which does not appear when weak-doublet mirrors of the type introduced in [1] are used. Even though these could *a priori* pose problems with the S parameter, it was recently shown [1] that vertex corrections could alleviate these effects. It is further shown that the most probable symmetry-breaking channel is the $SU(4)_{\rm PS} \times SU(2)_{\rm R} \longrightarrow SU(3)_{\rm C} \times U(1)_{\rm Y}$. This breaking channel corresponds to a unification scale $\Lambda_{\rm GUT}$ small enough to suggest that detection of proton decay could soon be experimentally accessible.

Present bounds on proton decay further indicate that no more fermion generations are very probable at low scales, since then, even though unification would still be possible, the unification coupling would be too large. Furthermore, the different running of the $SU(3)_{\rm C}$ and $SU(3)_{\rm 2G}$ gauge couplings due to the different fermion numbers which correspond to them provides a natural and very interesting explanation of the hierarchy between the QCD scale and the weak scale, i.e., approximately the scale where the generation interactions $SU(3)_{\rm 2G}$ become strong.

The unification investigation conducted also makes apparent a problem in this theory having to do with the generation of lepton masses. In particular, the Pati–Salam scale $\Lambda_{\rm PS}$ is found to be too large to allow quark masses to be fed down to leptons via effective four-fermion operators associated to the $SU(4)_{\rm PS}$ breaking. If one does not want to use a fundamental Higgs mechanism to break the generation group at the TeV scale, a solution to this problem would be a strong $U(1)_{\rm G}$ at the TeV scale [1]. To avoid a Landau pole to the corresponding gauge coupling, the group $U(1)_{\rm G}$ has to be embedded soon into a larger nonabelian group, like $SU(4)_{\rm 2G}$.

However, if one insists on unifying the generation coupling with the rest of the gauge couplings, the solution above is unfortunately not viable, since as was noted in Sect. 2.4, the $SU(4)_{\rm 2G}$ coupling runs too fast to unify with the other gauge couplings. The group $SU(4)_{\rm 2G}$ therefore has to be broken at the unification scale, and the $U(1)_{\rm G}$ coupling at low energies is consequently very weak. A way out for lepton-mass generation could in principle be the existence of gauge-invariant operators generated beyond tree level which feed down quark masses to leptons.

As has already been stressed, in the unification analysis presented, several effects are neglected. These are related to (i) unification threshold effects, (ii) the Higgs content needed to break the $SU(4)_{\rm PS}$ and $SU(2)_{\rm R}$ symmetries, (iii) two- and higher-loop contributions to the

β functions, (iv) the fact that the mirror fermions are taken to be all degenerate in mass, and (v) the $SU(3)_{2G}$ becoming strongly coupled at around 1 TeV, something that could influence the rest of the couplings. It is not expected, however, that these effects would spoil the qualitative results of the analysis above. Unification could still be achieved if these effects were correctly taken into account, since one has the freedom to adjust the scales Λ_{M} and Λ_{PS} without influencing considerably the unification scale Λ_{GUT}. This is particularly true for the favored possibility presented in Fig. 1, since the proximity of the scales Λ_{PS} and Λ_{GUT} does not leave room for large adjustments.

One could of course claim that the freedom to adjust Λ_{PS} to achieve unification makes this exercise easier to complete and reduces the predictability of the theory by adding an extra free parameter. On the other hand, the most favored scenario described connects this scale with the breaking not only of $SU(4)_{\mathrm{PS}}$, but also of the $SU(2)_{\mathrm{R}}$ symmetry. The examples involving Majorana neutrinos presented above indicate, however, that this scale is expected to be several orders of magnitude smaller than the unification scale. This not only speaks against the idea that a desert reaches up to Λ_{GUT}, but is also consistent with the scenario analyzed here.

As has already been noted, the unification considerations above indicate a favored $SU(2)_{\mathrm{R}}$-breaking scale usually associated with the mass of heavy Majorana neutrinos in the context of the seesaw mechanism. This leads to the study of neutrino masses and mixings, in this framework and in connection with recent experimental results. It is found that to have the neutrino masses and mixings compatible with experiment and unification, heavier charged mirror fermions than the ones quoted in [1] might be needed, unless the heavy standard-model Majorana neutrinos are quite lighter than the scale where $SU(2)_{\mathrm{R}}$ breaks. Heavier mirror fermions imply, furthermore, not only a more difficult detection of their indirect effects, since their mixing with the standard-model fermions is smaller, but also a smaller need of fine-tuning of their masses [1]. It is also interesting to show that the above observation is perfectly consistent with a small S parameter, which is currently favored by electroweak precision tests. A small S parameter could furthermore be an indication that the lightest mirror neutrino, i.e., the field denoted as $\nu_{1\mathrm{R}}^{\mathrm{M}}$, is so light that it could lie just beyond the reach of present high-energy collider experiments.

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